Math Virtual Learning

Precalculus with Trigonometry

Students will verify Trigonometric Identities using the Double Angle Identities

May 1st, 2020

Precalculus with Trigonometry
Lesson: May 1st, 2020

Objective/Learning Target:
Students will verify Trigonometric Identities using the Double Angle Identities

Today's lesson will cover how to use the Double Angle Identities to verify trigonometric identities. A reminder that we covered verifying identities in the Friday, April 24th and Monday, April 27th lesson.

Let's Get Started!
Watch Video:
[Verifying Identities Using Double Angle Formula]

Double Angle Identities

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Example: Verify that $\cot x \sin 2x = 1 + \cos 2x$ is an identity.

$$\cot x \sin 2x = \frac{\cos x}{\sin x} \sin 2x$$ Quotient identity

$$= \frac{\cos x}{\sin x}(2 \sin x \cos x)$$ Double-angle identity

$$= 2\cos^2 x$$

$$= 1 + \cos 2x$$ $\cos 2x = 2\cos^2 x - 1 \Rightarrow 2\cos^2 x = 1 + \cos 2x$

Example

Ex 1. Verify the following identity.

$$(\sin x + \cos x)^2 = 1 + \sin 2x$$

$= (\sin x + \cos x)(\sin x + \cos x)$

$= \sin^2 x + \sin x \cos x + \cos x \sin x + \cos^2 x$

$= \sin^2 x + 2 \sin x \cos x + \cos^2 x$

$= \sin^2 x + \cos^2 x + 2 \sin x \cos x$

$= 1 + 2 \sin x \cos x = 1 + \sin 2x$

Pythagorean Identity:

$\sin^2 x + \cos^2 x = 1$

Double Angle Formula for Sine:

$\sin 2x = 2 \sin x \cos x$

Practice

Use the Double Angle Identities to verify the following equations.

1 Verify the identity $1 - \cos 2x = \tan x \sin 2x$.

2 Prove that $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$.

3 Prove the identity: $\tan 2x + \frac{1}{\cos 2x} = \frac{\sin x + \cos x}{\cos x - \sin x}$

Practice - ANSWERS

1

$$1-\cos 2x = \tan x \sin 2x$$

$$1-\cos 2x = \left(\frac{\sin x}{\cos x}\right)(2\sin x \cos x)$$

$$1-\cos 2x = 2\sin^2 x$$

$$1-\left(1-2\sin^2 x\right) = 2\sin^2 x$$

$$2\sin^2 x = 2\sin^2 x$$

2

$$\begin{aligned}
\text{LHS} &= \frac{\sin\theta + 2\sin\theta\cos\theta}{1+\cos\theta+(2\cos^2\theta-1)} \\
&= \frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)} \qquad \text{(factorise)} \\
&= \frac{\sin\theta}{\cos\theta} \\
&= \tan\theta \\
&= \text{RHS}
\end{aligned}$$

3

$$\begin{aligned}
\text{LHS} &= \tan 2x + \frac{1}{\cos 2x} \\
&= \frac{\sin 2x}{\cos 2x} + \frac{1}{\cos 2x} \\
&= \frac{\sin 2x + 1}{\cos 2x} \\
&= \frac{2\sin x\cos x + \cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x} \\
&= \frac{(\cos x + \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)} \\
&= \frac{\cos x + \sin x}{\cos x - \sin x} \\
&= \text{RHS}
\end{aligned}$$

Additional Practice and Resources:

Additional Resource Videos:

Verifying Identities with Double Angle

Additional example of verifying with Double Angle

Additional Practice:

Verify with Double Angle Identities - Kuta

Try problems #13-16